Tiling of the PS1 Survey Area

The goal is to cover the survey area with cells that have a rectangular, preferably square shape in the tangent plane. The optimal layout will minimize the overlap of the cells.

Using declination zones to cover the 3π

Assume we have a square projection cell of size *a* degrees in the TAN projection. This corresponds to an angular size on the sky of θ, given by

$$a = 2\tan\frac{\theta}{2} \tag{1}$$

Furthermore, assume that we lay down the cells on constant declination zones, equally spaced over the circle. These zones should be separated such a way that the squares covering the adjacent zones do not leave gaps behind, preferably there is a little overlap. Let us assume that the zones are labeled with an integer number *n*, with *n*=0 at the equator. The centers of the cells will be at declinations δ_n. The number of cells at zone *n* is m_n. For the sake of simplicity let us assume that we are only dealing with the Northern hemisphere. This number must be chosen such that there is no gap between the cells.

Before we continue, we must compute the coordinates of the vertices of the cells, connected with great circles. In the *n*th zone, the midpoints of the lower and upper edges of the cells will be at declinations $\delta_n \pm \theta/2$. Thus the constant declination small circles connecting the top two vertices will be inside the cell, while the small circle connecting the two bottom vertex points will be outside the cell. Assume that the center of the cell is at $(\alpha, \delta) = (0, \delta_n)$.

Using the equations for the TAN projection, with the projection characterized by the normal vectors (**n**,**u**,**w**), denoting normal, up, and west, we get the unit vector for a point with (x,y) coordinates in the tangent plane as

$$\mathbf{r} = \frac{\mathbf{n} + y\mathbf{u} + x\mathbf{w}}{\sqrt{1 + x^2 + y^2}} \tag{2}$$

The three normal vectors are defined with (α, δ) as

$$\mathbf{n} = \begin{pmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -\sin\delta\cos\alpha \\ -\sin\delta\sin\alpha \\ \cos\delta \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} -\sin\alpha \\ \cos\alpha \\ 0 \end{pmatrix}. \tag{3}$$

The coordinates for the vertices are $x,y = \pm a/2$. The right ascension of the vertices can be calculated as

$$\tan\alpha_v = \frac{x}{1 - y\tan\delta} = \frac{\tan(\theta/2)}{1 \mp \tan\delta\tan(\theta/2)}, \tag{4}$$

where the - sign belongs to the upper edge, and the + sign is at the bottom edge. The highest declination of the bottom edge is

$$\delta_{-} = \delta_n - \theta / 2 \ . \tag{5}$$

The circumference of the lower small circle needs to be evenly subdivided between the cells. The circumference is $2\pi \cos\delta_{-}$, thus

$$m_n = \left\lfloor \frac{2\pi \cos \delta_{-}}{\theta} \right\rfloor + 1 \tag{6}$$

where the […] means the floor of the expression. The small circle at declination δ_n needs to be split into m_n equal segments. In right ascension this will yield an equal spacing of

$$\alpha_n = \frac{2\pi}{m_n} . \tag{7}$$

In order to have a full overlap between the adjacent zones, we have to choose the spacing such, that the lowest points on the top edge of zone *n* are at the same declination as the highest points of the bottom of the zone above (the midpoints of the lower edge). The lowest point on the upper edge of a given zone is given by the intersection of the top edges of adjacent cells.

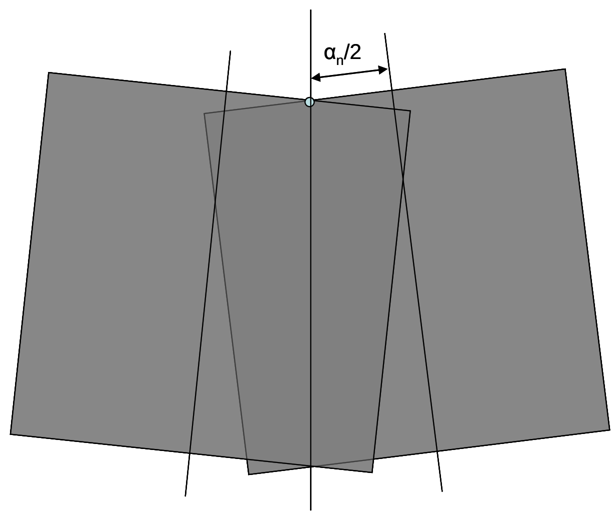

Figure 1. The point with the lowest declination in a zone is given by the intersection of two adiacent cells.

What is the declination of this point? The upper edge is given by

$$\frac{(\mathbf{r.u})}{(\mathbf{r.n})} = \frac{a}{2} . \tag{8}$$

If we consider the center of the cell to be at $(0, \delta_n)$, this gives the constraint that any point of the upper edge at right ascension α will have a declination δ_+ given by

$$\tan \delta_{+} = \tan(\delta_n + \theta / 2) \cos \alpha \tag{9}$$

Thus the declination of the critical point on the lower zone needs to be matched with the top value along the bottom edge of the next zone (the low midpoint).

$$\tan(\delta_{n+1} - \theta/2) = \tan(\delta_n + \theta/2)\cos(\alpha_n/2) \tag{10}$$

This determines the sequence of the zones. Figure 2 illustrates this scheme in an Aitoff projection and Figure 3 shows the worst case fields around the pole in a stereographic projection. The overhead on this scheme is 7.5% but almost nothing at the equator.

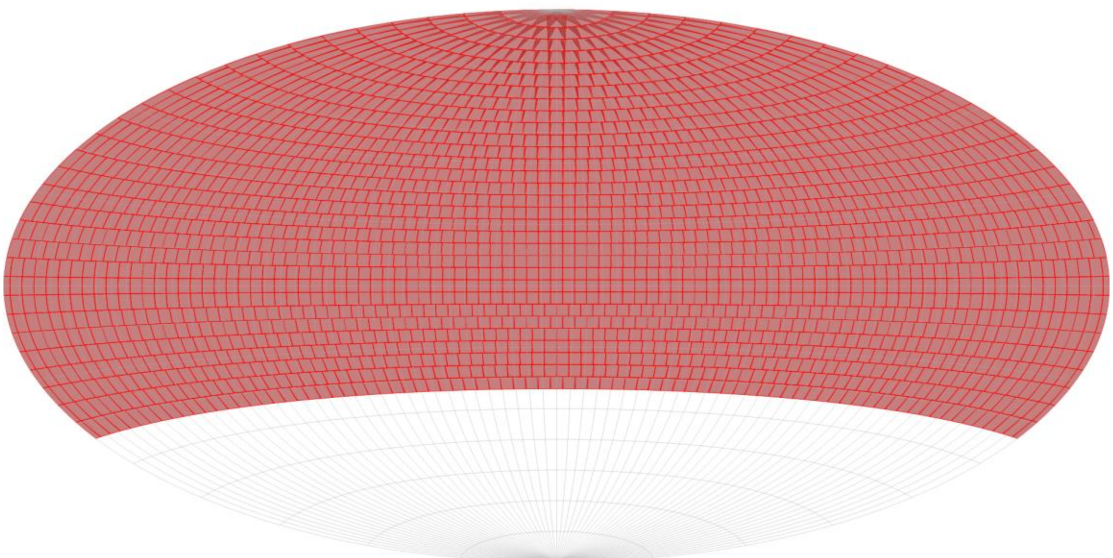

Figure 2. Aitoff projection of optimal bands with 7.5% overlap for the 3π

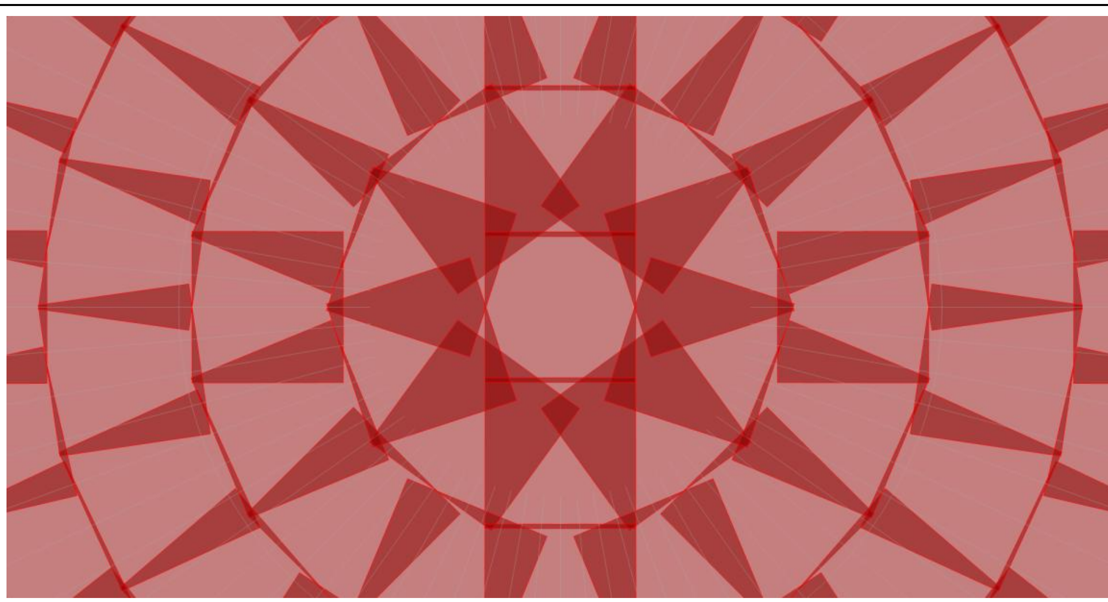

Figure 3. Fields with the largest overlaps at the pole